Majorana in nanowires
Lecture I (braiding)

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a particle that is its own antiparticle

\[ \gamma^\dagger = \gamma \]

an equal superposition of electron and hole
Majorana fermion $\gamma^+ = \gamma$

single electron, hole

$c^+, c$

c  =  $\gamma_1 + i\gamma_2$

c$^+$  =  $\gamma_1 - i\gamma_2$
Solid Theoretical Foundation

Toy model: 

\[ H_{\text{Kitaev}} = \sum_i \left( -t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu(c_i^\dagger c_i - \frac{1}{2}) + \Delta c_i c_{i+1} + h.c. \right) \]

A. Kitaev, Physics Uspekhi (2001)

First proposal based on existing materials – topological insulators

L. Fu and C. Kane PRL 2008

Semiconductor nanowire proposals:

\[ \mathcal{H} = \left[ \frac{p^2}{2m} - \mu(y) \right] \tau_z + u(y)p \sigma_z \tau_z + B(y)\sigma_x + \Delta(y)\tau_x. \]

Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010

\[ H_{\text{Kitaev}} = \sum_i \left( -t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu (c_i^\dagger c_i - \frac{1}{2}) + \Delta c_i c_{i+1} + \text{h.c.} \right) \]
Spinful case – no Majorana
**Majorana recipe:**

1. One-dimensional wire
2. Spin-orbit interaction
3. Superconductivity
4. Magnetic field

\[ \mathcal{H} = \frac{p^2}{2m} - \mu(y) \tau_z + u(y)p \sigma_z \tau_z + B(y)\sigma_x + \Delta(y)\tau_x. \]

\[ \gamma^+_1 = \gamma_1 \]
\[ \gamma^+_2 = \gamma_2 \]

Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010
Topological Transition

$E_Z < \Delta$

$E_Z = \Delta$

$E_Z > \Delta$

Trivial Superconductor

“positive gap”

Majorana

“zero gap”

Topological Superconductor

“negative gap”
Tunneling into a Majorana bound state: Resonant Andreev current!

Zero bias peak, $2e^2/h$ conductance
Theory: Law, Lee & Ng PRL 2009
Claimed by Zhang et al Nature 2019 (but I see problems with that paper.)
Majorana recipe:

Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010

1. Nanowire
2. Spin-orbit interaction
3. Superconductivity
4. Magnetic field

InSb nanowires: length 3 μm, diameter 100 nm
Plissard et al., Nano Letters 2012
Induced superconducting gap is one measure of topological protection

Hard gap realized at zero field, Generically ‘softens’ when field is applied to induce Majorana

Z. Su, A. Zarassi et al PRL 2018
Non-abelian anyons!
2-Majorana system is a fermion box

\[ \star + i \star \]

\[ \star - i \star \]
Qubit flip (braiding) in a nanowire T-junction
No need to move anything in space (except information)
Majorana qubit readout: fusion and parity measurement

$$|1\rangle \quad c^\dagger = \gamma_1 - i\gamma_2$$

$$|0\rangle \quad c = \gamma_1 + i\gamma_2$$
Even-odd effect in superconducting islands

Adding single electrons to an island of superconductor costs extra energy $\Delta$ (only observed in aluminum islands)

Adding single electrons to a **Majorana superconductor island** costs no extra energy (the island remains a superconductor!)

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**Eiles, Martinis, Devoret PRL 1993**
Readout Majorana qubit by detecting charge on an island with a transmon qubit
Hassler NJP 2011

with quantum dots
Karzig PRB 2017
4π Josephson effect

\[ \phi \neq \pi \]

\[ \phi = \pi \]

1e-periodic Josephson effect

Dynamical signatures (Shapiro, Josephson radiatio) can manifest without Majorana

Lutchyn, Sau, Das Sarma PRL 2010
Two qubits in one device
Readout and initialization
No braiding
CNOT gate:
Braidonium Qubit
with John Stenger, Michael Hatridge, David Pekker – PRB 2019

- Conventional SC
- P-Wave SC
- Uncovered SM
- Flux Coil
- Electrostatic Gates
- MBS

\[ \left| a_0 \right|^2 \]

\[ \log \tau \alpha \]

\[
\begin{array}{c|cccc}
\alpha/E_M & 0.01 & 0.00037 & 0.0014 & 0.0005 \\
\end{array}
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