Basic Concepts in Quantum Information:
Quantum Memories
Quantum Error Correction

Theory
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Experiment
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http://quantuminstitute.yale.edu/
Lecture I: Introduction to circuit QED

Lecture II: Quantum state engineering in the strong-dispersive limit: Cat States

Lecture III: Quantum Error Correction with Cat States

Experiment:

Theory:

‘New class of error correction codes for a bosonic mode’
Remarkable Progress in Coherence

Progress = 10 x every 3 years!
Girvin’s Law:

There is no such thing as too much coherence.

We need quantum error correction!
Quantum Error Correction

N qubits have errors $N$ times faster. Maxwell demon must overcome this factor of $N$ – *and not introduce errors of its own!*
The logical qubit should have longer coherence time than the best of its physical physical components.
Quantum Error Correction

QEC is an emergent collective phenomenon: adding N-1 worse qubits to the 1 best qubit gives an improvement!
Are We There Yet?

“Age of Qu. Error Correction.”
“Age of Quantum Feedback”
“Age of Measurement”
“Age of Entanglement”
“Age of Coherence”

Goal of next stage: reaching “break-even” point for error correction

“We” are ~ here (also ions, Rydbergs, q-dots, …)

M. Devoret and RS, Science (2013)
Information is **physical**.

Information is stored in, and transmitted by, **physical systems**

e.g., stored in physical position of a switch; transmitted by light
Classical bits

Information is stored in, and transmitted by, physical systems. There exist (only) two possible encodings.

“0”

“1”

“1”

“0”
Classical bits

Information is stored in, and transmitted by, physical systems. There exist (only) two possible encodings.

Transformation law between the two possible encodings:

\[ \text{NOT} \]

N.B. The information content (Shannon entropy) Bob receives is not affected by which decoding he uses. Bob can apply \text{NOT} to the decoded message \textit{post facto}. 
Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, physical systems.

Goal is faithful transmission of information across space and/or time.

Because there are only two encodings, there is only one possible error: bit flip.
Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, **physical systems**

**Goal is faithful transmission of information across space and/or time.**

**Essential tools:** Error correcting codes using ancilla bits
Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, physical systems

Alice 0110111010  →  Bob 0110111010

Alice 0110111010  →  Alice 0110111010

Essential tools:
Error correcting codes using ancilla bits

Example: majority rule replication codes

\[ 0 \rightarrow 000 \]
\[ 1 \rightarrow 111 \]
Quantum bits (‘qubits’)

Quantum information is stored in the physical (superposition) states of a quantum system:
- atoms, molecules, ions, superconducting circuits, photons, mechanical oscillators, …
Quantum Information is Paradoxical
Is quantum information carried by waves or by particles?

YES!
Is quantum information analog or digital?

YES!
Quantum information is digital:

Energy levels of a quantum system are discrete.

We use only the lowest two.

Measurement of the state of a qubit yields (only) 1 classical bit of information.

Excited state $1 = |e\rangle = |\uparrow\rangle$

Ground state $0 = |g\rangle = |\downarrow\rangle$
Quantum information is analog:

A quantum system with two distinct states can exist in an infinite number of physical states (‘superpositions’) intermediate between $\downarrow$ and $\uparrow$.

Bug:
We are uncertain which state the bit is in. Measurement results are truly random.

Feature:
Qubit is in both states at once, so we can do parallel processing. Potential exponential speed up.
Quantum information is **analog**:

A quantum system with two distinct states can exist in an infinite number of physical states (‘superpositions’) *intermediate* between $|\downarrow\rangle$ and $|\uparrow\rangle$.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right)|\uparrow\rangle$$

- $\theta = \text{latitude}$
- $\phi = \text{longitude}$

State defined by ‘spin polarization vector’ on Bloch sphere.
Quantum information is analog/digital:

Equivalently: a quantum bit is like a classical bit except there are an infinite number of encodings (aka ‘quantization axes’).

\[ Z = \pm 1 \quad \text{and} \quad Z' = \pm 1 \]

If Alice gives Bob a \( Z = +1 \), Bob measures:

\[ Z' = +1 \text{ with probability } P_+ = \cos^2 \frac{\theta}{2} \]

\[ Z' = -1 \text{ with probability } P_- = \sin^2 \frac{\theta}{2} \]

‘Back action’ of Bob’s measurement changes the state, but it is invisible to Bob.
What is knowable?

Consider just 4 states:

We are allowed to ask only one of two possible questions:

Does the spin lie along the Z axis? Answer is always yes! \((\pm 1)\)
Does the spin lie along the X axis? Answer is always yes! \((\pm 1)\)

BUT WE CANNOT ASK BOTH!
Z and X are **INCOMPATIBLE** OBSERVABLES
What is knowable?

We are allowed to ask only one of two possible questions:

Does the spin lie along the Z axis? Answer is always yes! \((\pm1)\)

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BUT WE CANNOT ASK BOTH!

Z and X are **INCOMPATIBLE** OBSERVABLES

**Heisenberg Uncertainty Principle**

If you know the answer to the Z question you cannot know the answer to the X question and vice versa.

(If you know position you cannot know momentum.)
Measurements 1.

State: Z
Result: quantum state is unaffected.

State: X
Result: \( \pm 1 \) randomly!
State is changed by measurement to lie along X axis.

Unpredictable result
Measurements 2.

State: $X$

Result: quantum state is unaffected.

State: $Z$

Result: $\pm 1$ randomly!
State is changed by measurement to lie along Z axis.

Unpredictable result
No Cloning Theorem

Given an unknown quantum state, it is **impossible** to make multiple copies.

Unknown state:

Guess which measurement to make --- if you guess wrong you change the state and you have no way of knowing if you did....
Given an unknown quantum state, it is **impossible** to make multiple copies

Big Problem:

Classical error correction is based on cloning! (or at least on measuring)

Replication code:

\[
\begin{align*}
0 & \rightarrow 000 \\
1 & \rightarrow 111
\end{align*}
\]

Majority Rule voting corrects single bit flip errors.
The Quantum Error Correction Problem

I am going to give you an unknown quantum state.

If you measure it, it will change randomly.

If it develops an error, please fix it.

*Miracle dictu:* It can be done!
Let’s start with classical error heralding.

Classical duplication code: $0 \rightarrow 00 \quad 1 \rightarrow 11$

Herald error if bits do not match.

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
<th># of Errors</th>
<th>Probability</th>
<th>Herald?</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00</td>
<td>0</td>
<td>$(1-p)^2$</td>
<td>Yes</td>
</tr>
<tr>
<td>00</td>
<td>01</td>
<td>1</td>
<td>$(1-p)p$</td>
<td>Yes</td>
</tr>
<tr>
<td>00</td>
<td>10</td>
<td>1</td>
<td>$(1-p)p$</td>
<td>Yes</td>
</tr>
<tr>
<td>00</td>
<td>11</td>
<td>2</td>
<td>$p^2$</td>
<td>Fail</td>
</tr>
</tbody>
</table>

And similarly for 11 input.
Using duplicate bits:
- lowers channel bandwidth by factor of 2 \( (\text{bad}) \)
- lowers the fidelity from \( (1 - p) \) to \( (1 - p)^2 \) \( (\text{bad}) \)
- improves unheralded error rate from \( p \) to \( p^2 \) \( (\text{good}) \)

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And similarly for 11 input.
Quantum Duplication Code
No cloning prevents duplication

\[ U(\alpha \downarrow + \beta \uparrow) \otimes \downarrow = (\alpha \downarrow + \beta \uparrow) \otimes (\alpha \downarrow + \beta \uparrow) \]

Proof of no-cloning theorem:
\( \alpha \) and \( \beta \) are unknown; Hence \( U \) cannot depend on them.
No such unitary can exist if QM is linear.
Q.E.D.
Don’t clone – entangle!

\[ U \left( \alpha |\downarrow\rangle + \beta |\uparrow\rangle \right) \otimes |\downarrow\rangle = \alpha |\downarrow\rangle |\downarrow\rangle + \beta |\uparrow\rangle |\uparrow\rangle \]

Quantum circuit notation:

\[ U = \text{CNOT} \]

\[ \alpha |\downarrow\rangle |\downarrow\rangle + \beta |\uparrow\rangle |\uparrow\rangle \]
Heralding Quantum Errors

\[ Z_1, Z_2 = \pm 1 \]

Measure the Joint Parity operator:

\[ \Pi_{12} = Z_1 Z_2 \]

\[
\begin{align*}
\Pi_{12} |\uparrow\rangle |\uparrow\rangle &= + |\uparrow\rangle |\uparrow\rangle \\
\Pi_{12} |\downarrow\rangle |\downarrow\rangle &= + |\downarrow\rangle |\downarrow\rangle \\
\Pi_{12} |\uparrow\rangle |\downarrow\rangle &= - |\uparrow\rangle |\downarrow\rangle \\
\Pi_{12} |\downarrow\rangle |\uparrow\rangle &= - |\downarrow\rangle |\uparrow\rangle \\
\end{align*}
\]

\[ \Pi_{12} (\alpha |\downarrow\rangle |\downarrow\rangle + \beta |\uparrow\rangle |\uparrow\rangle) = + (\alpha |\downarrow\rangle |\downarrow\rangle + \beta |\uparrow\rangle |\uparrow\rangle) \]

\[ \Pi_{12} = -1 \text{ heralds single bit flip errors} \]
Heralding Quantum Errors

$$\Pi_{12} = Z_1 Z_2$$

Not easy to measure a joint operator while not accidentally measuring individual operators!

(Typical ‘natural’ coupling is $M_Z = Z_1 + Z_2$)

$$|\uparrow\rangle|\uparrow\rangle \text{ and } |\downarrow\rangle|\downarrow\rangle$$ are very different,

yet we must make that difference invisible

But it can be done if you know the right people...
Example of error heralding:

\[ |\Psi\rangle = \alpha |\downarrow\rangle |\downarrow\rangle + \beta |\uparrow\rangle |\uparrow\rangle \]

Introduce single qubit error on 1 (over rotation, say)

\[ e^{i\frac{\theta}{2}X_1} |\Psi\rangle = \cos \frac{\theta}{2} |\Psi\rangle + i \sin \frac{\theta}{2} X_1 |\Psi\rangle \]

Relative weight of \( \alpha, \beta \) is untouched.

Probability of error: \( \sin^2 \frac{\theta}{2} \)

If no error is heralded, state collapses to \( |\Psi\rangle \)

and there is no error!
Heralding Quantum Errors

Example of error heralding:

\[ |\Psi\rangle = \alpha |\downarrow\rangle|\downarrow\rangle + \beta |\uparrow\rangle|\uparrow\rangle \]

Introduce single qubit rotation error on 1 (say)

\[ e^{i\frac{\theta}{2}X_1} |\Psi\rangle = \cos \frac{\theta}{2} |\Psi\rangle + i \sin \frac{\theta}{2} X_1 |\Psi\rangle \]

Relative weight of \(\alpha, \beta\) is untouched.

Probability of error: \(\sin^2 \frac{\theta}{2}\)

If error is heralded, state collapses to \(X_1 |\Psi\rangle\) and there is a full bitflip error. We cannot correct it because we don't know which qubit flipped.
Heralding Quantum Errors

Quantum errors are continuous (analog!).

But the detector result is discrete.

The measurement back action renders the error discrete (digital!)
  – either no error or full bit flip.
Correcting Quantum Errors

Extension to 3-qubit code allows full correction of bit flip errors

\[ |\Psi\rangle = \alpha |\downarrow\rangle|\downarrow\rangle|\downarrow\rangle + \beta |\uparrow\rangle|\uparrow\rangle|\uparrow\rangle \]

\[ \Pi_{12} = Z_1Z_2 \text{ and } \Pi_{32} = Z_3Z_2 \]

Provide two classical bits of information to diagnose and correct all 4 possible bitflip errors:

\[ I, X_1, X_2, X_3 \]
**Correcting Quantum Errors**

Extension to 5, 7, or 9-qubit code allows full correction of ALL single qubit errors

- $I$ (no error)
- $X_1, \ldots, X_N$ (single bit flip)
- $Z_1, \ldots, Z_N$ (single phase flip; no classical analog)
- $Y_1, \ldots, Y_N$ (single bit AND phase flip; no classical analog)

For $N=5$, there are 16 errors and 32 states

$$32 = 16 \times 2$$

*Just enough room to encode which error occurred and still have one qubit left to hold the quantum information.*
The cat code and hardware-efficiency


(Slides courtesy R. Schoelkopf)
Cat-codes Can Be “Hardware-Efficient”


- Advantages include:
  - Long lifetimes*: $Q \sim 10^8$, $\tau_c \sim 0.005$ sec
  - Large Hilbert space
  - One dominant error channel: single photon loss

\[ \kappa = \frac{1}{\tau_c} = \frac{\omega}{Q} \]

Our approach:
- Cavity is the memory
- One error syndrome

What is the best encoding scheme?

*Reagor et al., arXiv 1508.05882

earlier ideas: Gottesman, Kitaev & Preskill, PRA 64, 012310 (2001)
Redundant Encoding in Cat States

Hardware-Efficient Code:
Corrects for single photon loss

Logical code words in cavity:

\[ |W_1\rangle = |C_\alpha^+\rangle = \mathcal{N}(|\alpha\rangle + |\alpha\rangle) \]
\[ |W_2\rangle = |C_{i\alpha}^+\rangle = \mathcal{N}(|i\alpha\rangle + |i\alpha\rangle) \]

Map qubit state to cavity:

\[ c_g |g\rangle + c_e |e\rangle \Rightarrow c_g |W_1\rangle + c_e |W_2\rangle \]

Store a qubit as a superposition of two cats of same parity*

*Leghtas, Mirrahimi, et al., PRL 111, 120501, (2013)
Redundant Encoding in Cat States

\[ |C^+_\alpha\rangle = |\alpha\rangle + |\alpha\rangle \]

**Error Syndrome:** All logical states eigenstates of parity

\[ |C^+_\alpha\rangle + |C^+_{i\alpha}\rangle \]

\[ +X_c \]

\[ +Y_c \]

\[ |C^+_\alpha\rangle + i|C^+_{i\alpha}\rangle \]

\[ |C^+_{i\alpha}\rangle = |i\alpha\rangle + |-i\alpha\rangle \]
Redundant Encoding in Cat States

\[ |C^-_\alpha\rangle = |\alpha\rangle - |-\alpha\rangle \]

**Error Syndrome:**
All logical states eigenstates of parity

**Even or Odd basis:**
Cat code basis can be of even or odd parity

\[ |C^-_\alpha\rangle + |C^-_{i\alpha}\rangle \]

\[ |C^-_{i\alpha}\rangle = |i\alpha\rangle - |-i\alpha\rangle \]

\[ +Z_c \]

\[ +X_c \]

\[ +Y_c \]
Error model for cavity: Damping into a cold bath

(Exact) Kraus representation (POVM) organized by number of photon losses (detector ‘clicks’). SHO is a special case: time of clicks unimportant, only total number of clicks counts:

$$\rho(t) = \sum_{\ell=0}^{\infty} E_\ell \rho(0) E_\ell^\dagger$$

$$E_\ell \equiv \sqrt{\gamma_\ell} e^{-\frac{\kappa}{2} \hat{n}_t} a^\ell, \quad \gamma_\ell \equiv \sqrt{\frac{(1-e^{-\kappa t})^\ell}{\ell!}}$$

$$\sum_{\ell=0}^{\infty} E_\ell^\dagger E_\ell = \hat{I}$$

‘New class of error correction codes for a bosonic mode’
Coherent states have special properties under the Kraus operations:

$$E_\ell |\alpha\rangle = \sqrt{\gamma_\ell} \alpha^\ell \left| e^{-\frac{\kappa t}{2}} \alpha \right\rangle$$

\[
|\alpha\rangle \rightarrow \frac{E_\ell |\alpha\rangle}{\langle \alpha | E_\ell^\dagger E_\ell |\alpha\rangle^{1/2}} = \left( \frac{\alpha}{|\alpha|} \right)^\ell \left| e^{-\frac{\kappa t}{2}} \alpha \right\rangle
\]

State is (pseudo) invariant! (up to an important phase).
Effect of photon loss on code words

\[ a |W_1\rangle = a(|\alpha\rangle + |-\alpha\rangle) \rightarrow (|\alpha\rangle - |-\alpha\rangle) \quad \text{(if } \alpha \text{ real)} \]
\[ a^2 |W_1\rangle = a^2 (|\alpha\rangle + |-\alpha\rangle) \rightarrow (|\alpha\rangle + |-\alpha\rangle) = |W_1\rangle \]

\[ a |W_2\rangle = a (|i\alpha\rangle + |-i\alpha\rangle) \rightarrow i (|i\alpha\rangle - |-i\alpha\rangle) \]
\[ a^2 |W_2\rangle = a^2 (|i\alpha\rangle + |-i\alpha\rangle) = (i)^2 (|i\alpha\rangle + |-i\alpha\rangle) = -|W_2\rangle \]

After loss of 4 photons cycle repeats:

\[ a^4 \left( \xi_1 |W_1\rangle + \xi_2 |W_2\rangle \right) \rightarrow \left( \xi_1 |W_1\rangle + \xi_2 |W_2\rangle \right) \]

We can recover the state if we know: \[ N_{\text{Loss}} \mod 4 \]
To Live & Die in a Cavity: What $\kappa$ Does

$$|\psi^0\rangle = c_g |\mathcal{C}_\alpha^+\rangle + i c_{ce} |\mathcal{C}_{i\omega\alpha}^+\rangle$$

$\alpha \rightarrow \alpha e^{\kappa t/2}$
The Experiment:
Implementing the cat code with superconducting circuits
QEC Setup: 2 Cavities + 1 Transmon Ancilla

H = \omega_s a^+a + \omega_q |e\rangle\langle e| - \chi_{sa} a^+a |e\rangle\langle e| - K_{ss} (a^+a)^2

Ancilla (transmon) readout fidelity ~ 99.5% in 400 ns

Thank you QLab
The Experiment: Implementing the cat code with superconducting circuits

1. Map ancilla qubit state into photonic cat code words
2. Monitor number parity jumps $M$
3. Conditioned on $M$, choose unitary $U_M$ to map cavity state back to ancilla qubit
4. Perform process tomography to determine fidelity
5. Compare error-corrected logical qubit performance to simple $(0,1)$ photon encoding (best uncorrectable code)
2016: First true Error Correction Engine that works

- Commercial FPGA with custom software developed at Yale
- Single system performs all measurement, control, & feedback (latency ~200 nanoseconds)
- 15% of the latency is the time it takes signals to move at the speed of light from the quantum computer to the controller and back!

A prototype quantum computer being prepared for cooling close to absolute zero.
Implementing a Full QEC System: Debugger View

(This is all real, raw data.)
Process Fidelity: Uncorrected Transmon

\[ \tau \approx 15 \mu s \]
\[ \alpha = \sqrt{2} \]

**Cats without QEC**

**Cats with QEC**

**Process Fidelity:**

- System’s Best Component Still keep ~80% of data

- QEC – NO POST-SELECTION

Exclude results with heralded errors

**\( \tau \approx 560 \mu s \)**

**\( \tau \approx 290 \mu s \)**

**\( \tau \approx 320 \mu s \)**

**\( \tau \approx 130 \mu s \)**

**\( \tau \approx 15 \mu s \)**
Future directions:

- still higher Q cavities;

- cat codes on two cavities;
  logical operations for cat states in two cavities

- scaling to many cavities coupled by qubits

- 'kitten codes' [PRX 6, 031006 (2016)]

- 'cat pumping' to overcome amplitude decay
We are on the way!

“Age of Qu. Error Correction.”

“Age of Quantum Feedback”

“Age of Measurement”

“Age of Entanglement”

“Age of Coherence”

Have achieved goal of reaching “break-even” point for error correction

“We” are ~ here (also ions, Rydbergs, q-dots, …)

M. Devoret and RS, Science (2013)
When it comes to quantum mechanics you have to think different
A Few of the Team Members

Chen Wang
Phillip Reinhold
Matt Reagor
Teresa Brecht
Nissim Ofek
Jacob Blumoff
Luyan Sun
Brian Vlastakis
Kevin Chou
Chris Axline
Brian Vlastakis

Funding:
Chen Wang
Phillip Reinhold
Matt Reagor
Teresa Brecht
Nissim Ofek
Jacob Blumoff
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Leonid Glazman
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Eric Holland
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Luigi Frunzio

M. Mirrahimi
Z. Leghtas
Liang Jiang
Funding

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A few more....
Extra Slides
The Cat Code – Failure Modes

Failures
1. Double Errors
2. Uncorrectable Errors
3. Readout Errors
4. Ancilla Preparation
5. Undesired Couplings
6. Forward Propagation

Sources
1. Double Errors: cavity $a^2$
2. Uncorrectable Errors: cavity $a^\dagger$
3. Readout Errors: transmon $T_\phi$
4. Ancilla Preparation: transmon $\Gamma^\uparrow$
5. Undesired Couplings: cavity $a^\dagger^2 a^2$
6. Forward Propagation: transmon $T_1$

Expected Lifetime Gain
1. Double Errors: $x \times 125$
2. Uncorrectable Errors: $x \times 20$
3. Readout Errors: $x \times 25$
4. Ancilla Preparation: $x \times 1000$
5. Undesired Couplings: $x \times 2000$
6. Forward Propagation: $x \times 0.7$

Opt. Meas. Rate (1/25 μs)
- Max. Meas. Rate
  - $x \times 5$
  - $x \times 20$
  - $x \times 8$
  - $x \times 3$
  - $x \times 7$
  - $x \times 2$