LECTURE 1:  Monday, June 3

Phase-sensitive measurements on superconducting quantum materials and hybrid superconductor devices

Josephson physics and techniques useful for exploring superconductor materials and devices, focusing on probing unconventional superconductors and junctions

LECTURE 2:  Tuesday, June 4

S-TI-S Josephson junction networks: a platform for exploring and exploiting topological states and Majorana fermions

A specific device architecture that may support Majorana fermions and shows promise for manipulating them for quantum computation processes

Dale J. Van Harlingen  University of Illinois at Urbana-Champaign
## Significant role of SUPERCONDUCTIVITY in Quantum Information Science

| Quantum materials | Superconductors were the first and the most-studied many-body system and continue to be critical to materials research. Every time we think the field is fading out, new classes of superconductors emerge.  
\textit{\textit{e.g.}} HTSC, heavy fermion superconductors, pnictide superconductors, topological superconductors, twisted graphene superconductivity, ... |
<table>
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<tr>
<td>Quantum sensing</td>
<td>Superconductors and devices are legendary for their sensitivity as detectors and probes of exotic phenomena. This in part because of their required low temperature operation that has driven cryogenic physics and technology, and in part because of their intrinsic quantum nature. \textit{\textit{e.g.}} MRI, SQUIDs, quasiparticle mixers, axion detectors, ...</td>
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<td>Quantum computing and Quantum simulation</td>
<td>Conventional superconductor-based qubits (transmons, etc.) are one of the leading technologies for quantum simulation and quantum computing, and topological qubits based on Majorana fermions are one of the most explored for next-generation candidates.</td>
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<td>Quantum communications</td>
<td>Photons rule in this space, so one of the grand challenges in quantum information science is “transduction” --- the transfer of quantum state information from qubit platforms into telecommunication photons.</td>
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Phase-sensitive measurements on superconducting quantum materials and hybrid superconductor devices

Agenda

1. Superconductivity --- very little, but enough
2. Phase-coherence
3. Josephson effect
4. Phase-dynamics ---- the RSJ-model
5. Multiply-connected geometries --- SQUIDs and arrays
★ 6. Josephson interferometry
7. Applications to quantum materials
   A. Measuring the order parameter of unconventional superconductors
   B. Measuring the current-phase relation of Josephson junctions with unconventional barriers
Cryogenics – low temperature physics
1. Get rid of thermal motion
2. Observe onset of new phenomena

Superconductivity $\Rightarrow R=0$
+ thousands of metallic compounds and alloys

More illuminating: what materials are NOT superconducting?

Magnetic materials (Mn, Fe, Co, Ni)  Good metals (Cu, Ag, Au)
Conventional ("classic") superconductivity

BCS theory:
Bardeen, Cooper, Schrieffer (1957)

- **MECHANISM** = attractive phonon-mediated electron-electron interaction $\rightarrow$ Cooper pairing

- **GROUND STATE** = superfluid pair condensate

- **EXCITATIONS** = normal "quasiparticles" with an isotropic energy gap

\[ \Delta(k) = \Delta \]

Quasiparticle tunneling spectroscopy reveals the fully-formed energy gap

\[ \psi = n_s e^{i\phi} \]
High Temperature Superconductivity (1986)

Alex Müller  Georg Bednorz
IBM Zurich Research Laboratory

Woodstock of Music (1969)
3 days / 33 acts / 500,000 hippies

Woodstock of Physics (1987)
8 hours / 50 talks / 2500 physicists

"Our lives will be changed" ... this did not turn out as expected but is rather true for physicists who challenged our understanding of condensed matter physics and opened new opportunities for superconductor research.
Conventional ("classic") superconductivity

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\[
\Delta(k) = \Delta
\]

\[
\psi = n_s e^{i\varphi}
\]

Quasiparticle tunneling spectroscopy reveals the fully-formed energy gap

\[
\text{T}_c \text{ increased slowly from 4K to 23K over 75 years from 1911 to 1986}
\]
Phase Coherence in multiply-connected superconductors

requirement of single-valuedness of the condensate wavefunction

Phase constraint: \[ \oint \vec{\nabla} \phi = 2\pi n \]

Superconducting Ring

\[ \vec{V} \phi = \left( \frac{2m}{n_\text{sc}} \right) \vec{J}_s + \left( \frac{2e}{\hbar} \right) \vec{A} \]

\[ \vec{J}_s = 0 \quad \text{in center of SC} \]

Ring with Josephson Junction (rf SQUID)

SC: \[ \vec{V} \phi = \left( \frac{2m}{n_\text{sc}} \right) \vec{J}_s + \left( \frac{2e}{\hbar} \right) \vec{A} \]

JJ: \[ \Delta \theta = \theta_1 - \theta_2 \]

\[ \oint \vec{V} \phi = \left( \theta_1 - \theta_2 \right) + \left( \frac{2\pi}{\Phi_0} \right) \oint \vec{A} \cdot d\vec{l} = 2\pi n \]

\[ \Phi = n \left( \frac{\hbar}{2e} \right) = n\Phi_0 \]

\[ \Phi_0 = 2.07 \times 10^{-15} \text{ Wb} \]

\[ \Phi_0 = 20 \text{ G} \cdot \mu\text{m}^2 \]

flux quantization

gauge-invariant phase

magnetic flux

\[ \oint \vec{V} \phi = 2\pi n = \phi + 2\pi \left( \frac{\Phi}{\Phi_0} \right) \]
Josephson Effect

Josephson junction can be thought of as the “transistor” of superconducting electronics

50th Anniversary of the Josephson Effect

Josephson junction can be thought of as the “transistor” of superconducting electronics.
Josephson Effect

Cooper pair correlations enable a supercurrent in a tunnel junction that depends on the gauge-invariant phase difference across the junction:

\[ \phi = (\theta_1 - \theta_2) - \left( \frac{2\pi}{\Phi_0} \right) \int A \cdot d\ell \]

\[ I = I_c \sin \phi \quad \text{Josephson supercurrent} \]

\[ U = -E_J \cos \phi \quad \text{Josephson coupling energy} \]

where \( E_J = \frac{\hbar I_c}{2e} = \frac{I_c \Phi_0}{2\pi} \)

In general: \( I = I_c \cos \phi \) "current-phase relation"

**Phase dynamics --- phase winds according to the "Josephson relation"**

\[ \frac{d\phi}{dt} = \frac{2eV}{\hbar} \]

At constant \( V \):

\[ \phi(t) = \frac{2e}{\hbar} \int V dt = \frac{2eV}{\hbar} t \]

\[ I(t) = I_c \sin \left( \frac{2eV}{\hbar} t \right) = I_c \sin \left( 2\pi f_J t \right) \]

Josephson frequency \( f_J = \frac{2eV}{\hbar} \) (483.6 THz/V)

**Josephson standard volt defined by:**

483597.84841698 GHz/V
RSJ (Resistively-Shunted Junction) model

Model junction as a Josephson junction in parallel with a resistor and capacitance

\[ I = I_c \sin \phi + \frac{V}{R} + C \frac{dV}{dt} \]

\[ V = \frac{\hbar}{2e} \frac{d\phi}{dt} \]

\[ I = I_c \sin \phi + \frac{\hbar}{2eR} \frac{d\phi}{dt} + \frac{\hbar C}{2e} \frac{d^2\phi}{dt^2} \]

\[ \left( \frac{\hbar C}{2e} \right) \frac{d^2\phi}{dt^2} + \left( \frac{\hbar}{2eR} \right) \frac{d\phi}{dt} + \frac{\partial}{\partial \phi} (-I\phi - I_c \cos \phi) = 0 \]

"mass"  "damping"  "potential"

Josephson dynamics: “phase particle” moving in a tilted washboard potential

\[ I < I_c: \text{ static solution} \]
\[ \phi = \text{constant} \rightarrow V=0 \]

\[ I > I_c: \text{ dynamic solution} \]
\[ \phi \text{ evolves in time} \rightarrow V > 0 \]
\[ \text{voltage oscillates at the Josephson frequency} \]
Phase dynamics in the superconducting state

Equilibrium phase shifts
\[ \phi_0 = \arcsin \left( \frac{I}{I_c} \right) \]

Barrier height drops
\[ \Delta U = 2E_J \]

Josephson inductance
\[ I = I_c \sin \phi \quad \rightarrow \quad \frac{dI}{dt} = I_c \cos \phi \frac{d\phi}{dt} = \left( \frac{2eI_c}{\hbar} \cos \phi \right) V \quad \rightarrow \quad V = L_J \frac{dI}{dt} \]

Josephson plasma frequency
\[ \omega_p = \frac{1}{\sqrt{L_J C}} = \sqrt{\frac{2eI_c \cos \phi}{\hbar C}} \approx \sqrt{\frac{2eI_c}{\hbar C}} \sqrt{1 - \left( \frac{1}{I_c} \right)^2}^{1/4} \]

Small oscillation frequency in washboard potential well

Important for superconducting qubits by creating anharmonic potential that allows lifting degeneracy of harmonic oscillator states.
**Critical current -- escape of phase particle from the potential well**

Transition to the finite voltage state always occurs before $I_c$, the "thermodynamic critical current", due to fluctuations:

- **Thermal activation over the barrier**
- **Macroscopic Quantum Tunneling (MQT) through the barrier**

**Thermal activation rate:**

$$\Gamma_T = \left( \frac{\omega_p}{2\pi} \right) e^{-\Delta U/k_B T}$$

**MQT rate:**

$$\Gamma_Q = \left( \frac{\omega_p}{2\pi} \right) e^{-7.2 \left( \frac{\Delta U}{\hbar \omega_p} \right) \left( 1 + \frac{0.87}{\omega_p R_c} \right)}$$

MQT dominates when $\hbar \omega_p > 7.2 k_B T$

The numerical factors come from a WKB approximation for tunneling through the washboard potential barrier and damping corrections.
RSJ model ——current-biased

Shape of the I-V characteristic depends on the damping

Low damping (large R): $\beta_c > 1$

I-V is hysteretic

High damping (small R): $\beta_c < 1$

I-V is single-valued

“McCumber parameter”

$\beta_c = \frac{2\pi I_c R^2 C}{\Phi_0}$

For $I > I_c$, the junction switches abruptly to finite voltage and the supercurrent is sinusoidal and averages to zero

For $I > I_c$, the supercurrent is non-sinusoidal has a finite average and averages to zero

$V = R \sqrt{I^2 - I_c^2}$
**dc SQUID**

A dc SQUID consists of two junctions embedded in a superconducting loop

\[ I = I_{c1} \sin \phi_1 + I_{c2} \sin \phi_2 \]

\[ \phi_1 - \phi_2 + 2\pi \left( \frac{\Phi}{\Phi_0} \right) = 0 \]

**Symmetric SQUID** \( \rightarrow \) \( I_{c1} = I_{c2} = I_0 \)

\[ I_c = 2I_0 \left| \cos \left( \pi \frac{\Phi}{\Phi_0} \right) \right| \]

**Asymmetric SQUID** \( \rightarrow \) \( I_{c1} > I_{c2} \)

\[ I_c = \sqrt{(I_{c1} - I_{c2})^2 + 4I_{c1}I_{c2} \cos^2 \left( \pi \frac{\Phi}{\Phi_0} \right)} \]

![Graphs showing critical current modulation](image-url)
dc SQUID w/ inductance

A finite inductance modifies the SQUID characteristic because the circulating currents generate a flux in the loop that adds to the applied flux.

This is an example of “self-field effects” in which the fields from the Josephson currents cannot be ignored.

Inductive SQUID

\[
\beta = \frac{2LI_0}{\Phi_0} > 0
\]

\[
\Phi_{\text{loop}} = \Phi + L(I_1 - I_2) = \Phi + LJ
\]

\[ J = I_1 - I_2 = \text{circulating current} \]

\[ \Phi = \text{applied flux} \]

![Graph showing critical current vs. applied flux for different values of \( \beta \). The critical current is modulated and reduced as \( \beta \) increases.](image)

Dependence on \( \beta \)
dc SQUID operation

A dc SQUID biased at constant current exhibits a voltage modulation with applied magnetic flux (period in $\Phi_0$)

For highest field sensitivity, bias just above $I_0$
Josephson Effect in extended junctions

1. Local relation --- tunneling is highly-directional so the supercurrent depends on the phase difference at each location across the junction.

The supercurrent depends on the local CPR and the local gauge-invariant phase difference across the junction:

\[
\phi(y) = (\theta_1(y) - \theta_2(y)) - \left(\frac{2\pi}{\Phi_0}\right) \int_1^2 A \cdot d\ell
\]

\[
J(\phi(y)) = J_c(y) \text{cpr}(\phi(y)) \quad \text{“local current-phase relation”}
\]

2. Phase coherent --- phases at each location are related \(\Rightarrow\) interference.

Critical current:

\[
I_c(y) = \int_{-w/2}^{w/2} t \cdot J(y) dy
\]

3. Small-junction limit --- ignore self-field effects (no screening of field by tunneling currents).
Josephson Interferometry: response to a magnetic field

Phase coherence $\rightarrow$ magnetic field induces a phase variation:

$$\phi(y) = \phi_0 + \frac{2\pi}{\phi_0} \int_0^y dy' \lambda_m B(y')$$

Uniform magnetic field $\Rightarrow$ $\phi(y) = \phi_0 + \frac{2\pi}{\phi_0} \lambda_m B y$ $\rightarrow$ linear phase variation

Uniform junction $\Rightarrow$ $I_c = \max \int_{-w/2}^{w/2} dy \sin(\phi_0 + \frac{2\pi}{\phi_0} \lambda_m B y) \rightarrow$ Fourier transform

Fraunhofer diffraction pattern

Single-slit optical interference
Josephson Interferometry: what it can tell you

\[ I_c(\Phi) = \max \int_{-w/2}^{w/2} dy \, t \, J_c(y) \, cpr \left( \phi_0 + \phi_{op}(y) + \frac{2\pi}{\Phi_0} \int_0^y dy' \, \lambda_m B(y') \right) \]

- Critical current variation
- Magnetic field variations
- Gap anisotropy
  - Domains
  - Charge traps
- Current-phase relation
- Order parameter symmetry
- Non-sinusoidal terms
  - \( \pi \)-junctions
  - Exotic excitations
    - e.g. Majorana fermions
- Unconventional superconductivity
  - Flux focusing
  - Self-field from tunneling current
  - Trapped vortices
  - Magnetic particles
Josephson Interferometry: what it can tell you

\[ I_c(\Phi) = \max \int_{-w/2}^{w/2} \, dy \, t \, J_c(y) \, cpr \left( \phi_0 + \phi_{op}(y) + \frac{2\pi}{\Phi_0} \int_0^y \, dy' \, \lambda_m B(y') \right) \]

- **Critical current variation**
  - Gap anisotropy
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- **Current-phase relation**
  - Non-sinusoidal terms
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- **Order parameter symmetry**
  - Unconventional superconductivity

- **Magnetic field variations**
  - Flux focusing
  - Self-field from tunneling current
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Josephson Interferometry: critical current variations

Uniform junction

Narrow junction
dc SQUID

Wide junction
dc SQUID
Josephson Interferometry: more critical current variations

Asymmetric junctions
(magnitude)

Asymmetric junctions
(magnitude and width)

Three-junction SQUID
Josephson Interferometry: what it can tell you

\[ I_c(\Phi) = \max \int_{-w/2}^{w/2} dy \int_{-w/2}^{w/2} J_c(y) \text{cpr} \left( \phi_0 + \phi_{op}(y) + \frac{2\pi}{\Phi_0} \int_0^y dy' \lambda_m B(y') \right) \]

- **Critical current variation**
- **Magnetic field variations**
- **Gap anisotropy**
  - Domains
  - Charge traps
- **Current-phase relation**
  - Non-sinusoidal terms
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  - Exotic excitations e.g. Majorana fermions
- **Order parameter symmetry**
  - Unconventional superconductivity
- **Flux focusing**
  - Self-field from tunneling current
  - Trapped vortices
  - Magnetic particles
My Friends in Urbana

Theorists with new ideas

What unique phenomena can I probe with Josephson Interferometry

Experimentalists with new materials

Is it superconducting?

YES

Is it unconventional or exotic?

YES

Measure the pairing symmetry

NO

Go away – I don’t care

Can I make it into the barrier of a Josephson junction?

YES

Measure the current-phase relation

NO

Go away – I don’t care

When all you have is a hammer, everything looks like a nail
(For me, everything looks like a SQUID)
HTSC --- many exciting phenomena and mysteries

SCIENTISTS: new physics
ENGINEERS: new applications of superconductivity

1. Ceramics --- oxide materials
2. $T_c$ --- higher than could be explained by conventional BCS
3. Layered materials --- low dimensional effects and strong correlations
4. Unusual properties --- thermodynamics, transport, electrodynamics, …
5. Unusual doping dependence --- complicated phase diagram
6. Unusual vortex states and dynamics
7. UNCONVENTIONAL SUPERCONDUCTIVITY
Unconventional superconductivity

What does “unconventional” mean? Not BCS
- **Mechanism** other than phonon-mediated pairing
- **Symmetry** not s-wave ... exhibits anisotropy in phase and/or magnitude

1st indication: $\text{UPt}_3$ (heavy fermion) $\rightarrow$ two peaks in specific heat
1st confirmation: $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ (high-$T_c$ superconductor) $\rightarrow$ d-wave

Cuprate superconductors:

$\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ $T_c = 92K$

“d-wave”
Determining the Pairing Symmetry --- A Roadmap for Experimentalists

Cuprate candidates

$s$ vs. $d_{x^2-y^2}$

Magnitude measurements probe quasiparticles --- but can be masked or mimicked by impurities

Phase measurements give a distinct signature --- less susceptible to microscopic details

Complex order parameter $\Rightarrow$ broken time-reversal symmetry

phase shift $\delta \neq 0, \pi$
Evolution of the Corner SQUID Idea

B9 MRL
December 1991

DVH Group meeting
January 1992
Vortices with half magnetic flux quanta in "heavy-fermion" superconductors

V. B. Geshkenbein and A. I. Larkin
Landau Institute for Theoretical Physics, Academy of Sciences of U.S.S.R., Moscow, U.S.S.R.

A. Barone
Dipartimento di Fisica Nucleare Struttura della Materia e Fisica Applicata, Università di Napoli, Napoli, Italy
and Istituto di Cibernetica del Consiglio Nazionale delle Ricerche, Arco Felice (Napoli), Italy
(Received 23 June 1986)
Josephson interferometry: measuring the phase anisotropy

the corner SQUID

Unconventional SC single crystal

Conventional SC thin film loop

dc SQUID (Superconducting QUantum Interference Device) measures the phase shift inside the crystal between orthogonal directions

Josephson tunnel junctions tunneling selects direction in k-space

Wollman, Ginsberg, Leggett, Van Harlingen (1993)
Josephson interferometry --- measuring the phase shift between different directions

SC crystal

s-wave SC thin film loop

Josephson junctions

dc SQUID

single junction
The corner SQUID experiment

s-wave

\[ \Phi_0 \]

Critical current

Magnetic flux (\( \Phi_0 \))

-3 -2 -1 0 1 2 3

0.0 0.2 0.4 0.6 0.8 1.0

0

-3 -2 -1 0 1 2 3

d-wave

\[ \Phi_0 \]

Magnetic flux (\( \Phi_0 \))

-1.0 \( \Phi \) -0.5 \( \Phi \) 0.0 \( \Phi \) 0.5 \( \Phi \) 1.0 \( \Phi \) 1.5 \( \Phi \) 2.0 \( \Phi \)

100\( \mu \)m

Observations

edge SQUIDs

corner SQUIDs

Phase shift

0 1 2 3 4 5 6 7 8

The corner junction experiment

Further phase-sensitive measurements

**Tricrystal ring experiment**

*Kirtley, Tsuei (IBM)*

---

**π-SQUID**

*Hilgenkamp, Mannhart (Augsburg)*

---

Angles of the grain boundary junctions determines if there is a spontaneous circulating current.

Angles of the grain boundary junctions determines if there is a $\pi$-phase shift in the SQUID modulation.
**Paramagnetic Meissner Effect**

**Experiment:** enhanced magnetic flux in granular BSCCO composites

**Model:** spontaneous supercurrents in multiply-connected d-wave grains

---


Grain boundary junctions

Geometry for testing symmetry: 45°-asymmetric junction → facets sample different signs of the d-wave order parameter

Maximum $I_c$ not at $B=0$
Symmetric with respect to field polarity
S-wave would give Fraunhofer pattern

Cuprates--- all d-wave, all the time?

The compilation of experiments --- spectroscopic, thermodynamic, transport, and phase-sensitive --- indicate that all of the cuprates have $d_{x^2-y^2}$ symmetry

Many proposals for inducing alternative pairing symmetries but so far not seen -- tested vs. material, temperature, carrier doping, magnetic impurities, ...

Example: Fragility of unconventional superconductors

- **s-wave superconductor**: scattering does not affect superconductivity → “Anderson theorem”

- **unconventional superconductor**: scattering changes magnitude and phase of the order parameter
  - formation of zero-energy bound quasiparticle states
  - suppression of d-wave at interfaces and defects
  - onset of SC phases with complex order parameters at low temperature → broken time-reversal symmetry

Tested by phase-sensitive Josephson interferometry
Effect of onset of complex order parameter

Onset of secondary order parameter:
* increases B=0 critical current $\to$ node lifted
* breaks polarity symmetry $\to$ broken TRS

EXPERIMENT
No evidence for a change in the order parameter symmetry over a wide temperature range and with doping of non-magnetic and magnetic impurities:

Where haven't we looked? --- the strongly overdoped cuprates

- d-wave superconductivity nucleates in spatially inhomogeneous "puddles" which couple together via the Josephson effect
- Produces a globally "s-wave"-like pairing symmetry despite local d-wave in grains
- Overdoped LSCO crystals are a candidate

Optimal-doping

Overdoping

David Hamilton (Illinois)
Masaki Fujita (Tohoku)
Steve Kivelson (Stanford)
LSCO (x=0.25, Tc=15K) crystal, grown by the Fujita group, is oriented via XRD, cut/polished, then mounted onto a sapphire substrate.

Using dry-film photoresist masking, Josephson junctions are patterned onto a- and b- faces of the crystal.

Au (annealed) is used as the normal barrier.

Superconducting contacts are made with sputtered Nb.
Josephson interferometry diffraction patterns --- edge or corner

- Observe short-range field modulations features suggestive of grain boundary junctions --- indicates domains

- Also see broken field symmetry which could arise from complex order parameters or local magnetic field inhomogeneity

- We do not observe a s-wave state, but we cannot tell corner from edge diffraction patterns --- this may be regarded as definition of “s-wave -like”
Model system with $d$-wave domains and Wohlleben effects

Circulating currents from Wohlleben effect generates an inhomogeneous field that threads the junction
Model simulates both the rapid oscillations and the magnetic field asymmetry.
Growing Family of Unconventional Superconductors

**Cuprate superconductors**

$YBa_2Cu_3O_{7-\delta}$  $T_c = 95K$

![Cuprate structure](Image)

“$d$-wave”

**115 superconductors**

$\kappa-(BEDT-TTF)_2Cu[N(CN)_2]Br$

$T_c = 11.6K$

![115 superconductors structure](Image)

“$d$-wave”

**Organic superconductors**

$CeCoIn_5$

$T_c = 2.3K$

![Organic superconductors structure](Image)

“$d$-wave”

**Ruthenate superconductors**

$Sr_2RuO_4$

$T_c = 1.5K$

![Ruthenate structure](Image)

$p_x+ip_y$

**Heavy Fermion superconductors**

$UPt_3$

$T_{cA} = 0.50$

$T_{cB} = 0.45K$

![Heavy Fermion structure](Image)

$(k_x^2-k_y^2)k_z$

$(k_x+ik_y)^2k_z$
The Quest for Complex Superconductors

Heavy Fermion superconductors:

\[ \text{UPt}_3 \quad T_{cA} = 0.50 \quad T_{cB} = 0.45 \text{K} \]

\[ (k_x^2 - k_y^2) k_z \quad (k_x + i k_y)^2 k_z \]

Ruthenate superconductors:

\[ \text{Sr}_2\text{RuO}_4 \quad T_c = 1.5 \text{K} \]

\[ p_x + ip_y \]
Josephson interferometry of complex order parameters

Angle SQUID

$\delta = \text{phase shift}$

Angle junction

$\delta = \text{phase shift}$
Phase shifts from complex order parameters

Angle SQUID

\[ \delta = \text{phase shift} \]

Angle junction

Polarity asymmetry indicates broken time-reversal symmetry
New questions of pairing symmetry --- opportunities for Josephson interferometry

Current focus is on topological systems --- interplay of topology and superconductivity

Superconductor/Topological Insulator bilayer

Corner Junction experiment to test for proximity-induced p-wave symmetry

Materials exhibit superconductivity, ferromagnetism, Andreev reflection, gate-dependent surface supercurrents, hints of p-wave superconductivity

Testing order parameter symmetry via corner SQUID/junction experiments

(C. Kurter, A. Finck, E. Huemiller, and Y.-S. Hor)

Topological Superconductors
e.g. $\text{Cu}_x\text{Bi}_2\text{Se}_3$, $\text{Nb}_x\text{Bi}_2\text{Se}_3$, ...
What’s next? the growing list of systems in which to determine the pairing symmetry

S/TI bilayers
Doped-TIs --- topological superconductor candidates
FeTe$_{1-x}$Se$_x$
PrOs$_4$Sb$_{12}$
YPtBi
LAO/STO interface superconductors
Twisted bilayer graphene superconductors
CeCoIn$_5$
CeCu$_2$Si$_2$
UBe$_{13}$
Sr$_2$RuO$_4$
???